Math 163.1 **Pre-Calculus I** **Learner Outcomes**

Preliminary Notes

1. *Contemporary Precalculus* (3rd Edition) by Thomas W. Hungerford is the recommenced text for this course. No matter which text you use, **you are required to have a copy of this text for reference**. Furthermore, you should refer often to the content, because this text strongly prescribes and integrated approach to teaching mathematics.
2. The following outline relates to the Hungerford text in which there is a strong problem-solving component that depends on technology. The three main Learner Outcome components are I. Technology, II. Algebra, and III. Problem Solving. Notably, the third component is only a partial list of problem types, and you are urged to fully integrate a strong problem-solving component into your course.
3. Learners should learn to demonstrate (i) correctly translating a problem into mathematics; (ii) correctly using appropriate mathematics to solve the resulting equation(s), and (iii) correctly sorting appropriate solutions from those that are extraneous or physically inordinate.
4. Learners should learn to demonstrate appropriate problem-solving strategies. **This is the spirit of the Rule of Three: to employ an appropriate mix of accuracy and efficiency in the solution of problems by algebraic, numerical, and graphical techniques.** That is, students should learn to choose algebraic techniques when a problem is efficiently solved by these, and numeric or graphic techniques when algebraic solutions are impossible or inordinately difficult. Such choices are a part of learning elegance and efficiency.

Learner Outcomes

I. Technology Component. Using a hand-held graphing calculator, the learner will be able to **graphically** and **numerically** demonstrate:

A. How to find solutions by graphing for functions of the following types:
   1. rational functions with emphasis on polynomials
   2. exponential functions
   3. logarithmic functions
   4. functions involving radicals
   5. algebraic combinations of functions of the above types

B. How to choose a proper calculator window to display any of the above functions in order to identify
   1. roots
   2. points of non-definition (vertical asymptotes and holes)
   3. functional end behavior, including
      a) horizontal asymptotes
      b) oblique asymptotes
      c) growth without bounds
   4. extrema (maxima and minima)
   5. regions of functional increase and decrease
   6. inflection points (optional)
   6. intersections of functions

C. How to effectively curve-fit 2-dimensional data, using the following regression models:
   1. linear: $y = ax + b$
   2. quadratic: $y = a(x-b)^2 + c$
   3. cubic: $y = ax^3 + b$
   4. power: $y = ax^b$
   5. exponential: $y = ab^x$

6. logarithmic: $y = a\log_b(x) + c$
7. logistic (optional and modeled in the TI-83)

D. How to solve a system of two linear equations or inequalities
E. How to solve equations and inequalities involving absolute value
F. How to write a program for the quadratic formula to approximate complex roots
G. How to graph functional transformations including
   1. translations (in x and y directions)
   2. reflections
   3. scaling (dilations and contractions)
   4 combinations of the above transformations

H. How to graphically demonstrate symmetry about
   1. the x-axis
   2. the y-axis
   3. the line y=x, as related to inverse functions
   4. the origin

II. Algebraic Component. Using **algebraic** methods the learner will be able to demonstrate:

A. How to solve equations of the following types
   1. linear
   2. quadratic by completing the square and by solving the quadratic equation
   3. rational with emphasis on polynomials
   4. radical
   5. exponential
   7. logarithmic

B. How to represent symbolically operations with functions, including
   1. addition, subtraction, multiplication, and division
   2. composition
   3. the role of composition in inverse functions
   4. piecewise-defined functions

C. How to describe functions as in part A. by
   1. describing their domains and ranges
   2. restricting domains of relations, including inverse relations, to create functions
   3. using the vertical and horizontal line tests
   4. using the concept of one-to-one in conjunction with invertibility of a function.

D. How to symbolically represent functional transformations, including
   1. translations (in x and y directions)
   2. reflections about the x and y axes
   3. scaling (dilations and contractions)
   4. combinations of the above transformations

E. How to recognize and symbolically represent functional symmetry, including
   1. symmetry about the x and y axes
   2. symmetry about the origin
   3. symmetry about y=x as it relates to functional invertibility

E. How to use calculus-related concepts such as
   1. average rate of change including the
      a) secant line model
      b) difference quotient
   2. end behavior or asymptotic behavior

F. How to apply various important theorems and techniques for solving equations, including
   1. The Fundamental Theorem of Algebra

2 factor and remainder theorems
3. polynomial division algorithms

G. Given factorization of a function, how to identify the following
   1. roots
   2. points of no definition (vertical asymptotes and holes)
   3. functional end behavior, including
      a) horizontal and vertical asymptotes
      b) oblique asymptotes
      c) growth without bounds
   4. holes in functions (points of non-definition)
   5. extrema (maxima and minima)
   6. inflection points (optional)
   7. regions of functional increase and decrease
   8. intersections of functions

H. How to solve exponential equations using laws of exponents and laws of logarithms, including change of bases.

I. How to construct and compute the difference quotient.

III. Problem-Solving Component. Using the **Rule of Three**, that is algebraically, graphically, and numerically, the learner will demonstrate solutions to the following types of problems:

A. falling body problems using Newton's formula: $x = 16t^2 + v_o t + x_o$
B. mixture problems
C. maximization or minimization problems such as finding optimal surface area to contain volume
D. geometric dimension problems involving finding the exact dimension of a figure given some volume and surface area.
E. problems involving Distance = Rate x Time
F. compound interest and continuous compound interest models
G. average velocity and other averages by use of the Newton quotient.
H. problems involving the construction of split function models
I. problems involving marginal increases
J. revenue, principle, cost models involving two equations